

# Thick brane isotropization in the 5D anisotropic standing wave braneworld model

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**Abstract.** We study a smooth cosmological solution of the 5D anisotropic standing wave braneworld model generated by gravity coupled to a phantom-like scalar field. In this model the brane emits anisotropic waves into the bulk with different amplitudes along different spatial dimensions. We found a natural mechanism which isotropizes the braneworld, rendering a 3-brane with de Sitter symmetry embedded in a 5D de Sitter space-time for a wide class of initial conditions. The resulting thick geometrical braneworld (a de Sitter 3-brane) possesses a series of remarkable features. By explicitly solving the bulk field equations we are able to give a physical interpretation of the anisotropic dissipation: as the anisotropic energy on the 3-brane rapidly leaks into the bulk, through the nontrivial Weyl tensor components, the bulk becomes less isotropic.

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## 1. Introduction

Braneworld models involving large extra dimensions have been very useful in addressing several open questions in high energy physics [1,2], and astrophysics and cosmology (for reviews see [3–5]). For more than a decade it is clear that braneworlds possess a set of appealing features that encourage one to develop this research line further. Namely, braneworld models recast the hierarchy problem into a higher dimensional viewpoint, reveal a geometrical mechanism of dimensional reduction supported by a curved extra dimension, can trap various matter fields on the brane (including gauge bosons in some cases), take into account the self-gravity on the brane through its tension, give a realization of the AdS/CFT correspondence to lowest order, allow one to address the smallness of the cosmological constant, provide several cosmological backgrounds with consistent dynamics that lead to General Relativity results under suitable restrictions on their parameters, etc. Most of the braneworlds were realized as time independent field configurations. However, mostly within the framework of cosmological studies, there have been proposed different braneworld models that employ time-dependent metrics and matter fields (if any), as well as branes with tensions varying in time [6–14].

It is widely believed that our Universe started its evolution from a highly anisotropic state which lead to a highly symmetric state that we observe nowadays (according to the recent WMAP data CMB is isotropic with an accuracy of  $10^{-5}$  [15]). An interesting cosmological question concerns the dynamical mechanism that led the Universe to shed almost all of its anisotropy along time till the present epoch which is highly isotropic. A very popular mechanism among cosmologists is inflation: the Universe diluted away almost all of its anisotropy during a period of accelerating expansion in an early epoch. Of course, there are several other mechanisms that tried to explain this important phenomenon (see [12] and references therein).

When studying braneworlds generated by anisotropic backgrounds, one must have a complete solution to the bulk and brane field equations in order to consistently analyze the cosmological dynamics on the brane. Such a solution involves the knowledge of the bulk Weyl tensor which is felt on the brane by means of its projection [16]. It turns out that this is not an easy task to solve [9] and up to now, there are a very few cosmological anisotropic braneworlds that present a complete solution. In this context, there are many studies reported in the literature that made several assumptions about the Weyl term in the absence of knowledge of the full solution to the bulk metric [17,18]. The main difficulty here consists in finding anisotropic generalizations of the  $AdS_5$  space-time that are necessarily non-conformally flat and incorporate anisotropy on the cosmological brane. In this sense, it is relevant to propose new braneworld generalizations that attempt to describe more realistic cosmological braneworld models, or explore other aspects of higher-dimensional gravity which were not addressed till now by the known brane models.

Recently a braneworld model generated by 5D anisotropic standing gravitational waves minimally coupled to a phantom-like scalar field in the bulk was proposed [19,20].

In this framework, an alternative mechanism for localizing 4D gravity as well as for trapping matter fields [21], including gauge bosons [22], which usually are not localized on thin braneworlds, was found.

In this paper we consider a thick version of the braneworld model [19, 20] and slightly modify the metric *ansatz* by adding a scale factor that multiplies the spatial sector of the anisotropic 3-brane in the 5D metric. When exactly solving the bulk field equations the cosmological constant turns out to be positive, realizing a de Sitter braneworld instead of an Anti de Sitter one. By leaving the found solution evolve in time, the metric isotropizes and the corresponding scalar field disappears, leading to a recently proposed de Sitter braneworld model purely generated by curvature, i.e., by an interplay between the 4D and 5D cosmological constants [23]. It turns out that within this regular braneworld one is able to model the thick brane in a completely geometrical way, avoiding at all the use of scalar matter. In this model the effective 4D Planck mass is finite (and also depends on the Hubble parameter of the metric), the 4D gravity can be localized and the correction to Newton's law have been computed. As an extra bonus, a mass gap is displayed in the gravity spectrum of Kaluza-Klein (KK) excitations (a fact that fixes the energy scale at which these massive fluctuations can be excited and enables us to avoid difficulties when analyzing the traces of ultra-light KK excitations) without developing naked singularities as in the case of scalar thick brane configurations [24]. Another interesting feature of this thick braneworld model is that its 4D cosmological constant can be made as small as one desires without the need of fine-tuning it with the bulk cosmological constant, as it happens in the Randall-Sundrum type models [2].

## 2. The model and the complete solution

We start with a 5D action which describes gravity coupled to a non-self-interacting scalar phantom-like field [25], which depends on time and propagates in the bulk:

$$S_b = \int d^5x \sqrt{g} \left[ \frac{1}{16\pi G} (R - 2\Lambda) + \frac{1}{2} (\nabla\phi)^2 \right], \quad (1)$$

where  $G$  and  $\Lambda$  are 5D Newton and cosmological constants, respectively. To avoid the well-known problems of stability which occur with ghost fields we assume that the bulk phantom-like scalar field  $\phi$  does not couple to ordinary matter in the model.

The Einstein equations for the action (1) are written in the form:

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T + \frac{2}{3} g_{\mu\nu} \Lambda = -\partial_\mu \sigma \partial_\nu \sigma + \frac{2}{3} g_{\mu\nu} \Lambda, \quad (2)$$

where Greek indices refers to 5D and we have redefined the scalar field as  $\sigma = \sqrt{8\pi G} \phi$  in order to absorb the 5D gravitational constant.

Here we shall use the anisotropic metric *ansatz*:

$$ds^2 = e^{2A(w)} \left[ -dt^2 + a^2(t) (e^u dx^2 + e^u dy^2 + e^{-2u} dz^2) + dw^2 \right], \quad (3)$$

where  $a(t)$  plays the role of a cosmological scale factor,  $u(t, w)$  describes standing gravitational waves of [19, 20] and the warp factor  $A(w)$  is an arbitrary function of

the extra coordinate  $w$ , allowing for smooth solutions, and models a thick brane configuration in the spirit of [26]. This metric generalizes straightforwardly the thin brane metric *ansatz* of [19, 20], to the case when  $A(w) \neq |w|$  and a scale factor  $a(t)$  multiplies the 3D anisotropic spatial sector of the metric.

Thus, in this braneworld, the 3-brane possesses anisotropic oscillations which send a wave into the bulk (as in [27]), i.e. the brane is warped along the spatial coordinates  $x, y, z$  through the factor  $e^{u(t,w)}$ , depending on time  $t$  and the extra coordinate  $r$ . Moreover, these warped spatial coordinates are multiplied by a scale factor  $a(t)$  that allows them to evolve in time (expanding or contracting) as an extra feature. This model also joins previously constructed anisotropic braneworlds [8–13] which approached several cosmological issues like anisotropy dissipation during inflation [10], braneworld isotropization with the aid of magnetic fields [12] and localization of test particles [13, 21, 22]. Moreover, as a general feature it has been established that anisotropic metrics on the brane cannot be supported by static bulks [11, 12].

On the other hand, the phantom-like scalar field  $\sigma(t, w)$  obeys the Klein-Gordon equation on the background (3):

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \sigma) = \ddot{\sigma} + \frac{3\dot{a}}{a} \dot{\sigma} - \sigma'' - 3A'\sigma' = 0, \quad (4)$$

where overdots and primes stand for derivatives with respect to time and extra coordinates, respectively.

Under the *ansatz* (3) we can write the Einstein equations in the form:

$$\begin{aligned} R_{tt} &= -\frac{3}{2}\dot{u}^2 - 3\frac{\ddot{a}}{a} + A'' + 3A'^2 = -\frac{2}{3}e^{2A}\Lambda - \dot{\sigma}^2, \\ R_{xx} &= a^2 e^u \left[ \frac{1}{2} \left( \ddot{u} + 3\frac{\dot{a}}{a}\dot{u} - u'' - 3A'u' \right) + \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - A'' - 3A'^2 \right] \\ &= R_{yy} = \frac{2}{3}e^{2A+u}a^2\Lambda, \\ R_{zz} &= a^2 e^{-2u} \left[ -\ddot{u} - 3\frac{\dot{a}}{a}\dot{u} + u'' + 3A'u' + \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - A'' - 3A'^2 \right] \\ &= \frac{2}{3}a^2 e^{2A-2u}\Lambda, \\ R_{ww} &= -\frac{3}{2}u'^2 - 4A'' = \frac{2}{3}e^{2A}\Lambda - \sigma'^2, \\ R_{tw} &= -\frac{3}{2}\dot{u}u' = -\dot{\sigma}\sigma'. \end{aligned} \quad (5)$$

From the  $tw$ -component of the Einstein equations (5), it follows that the fields  $\sigma$  and  $u$  are related by

$$\sigma(t, r) = \sqrt{\frac{3}{2}} u(t, r), \quad (6)$$

up to a meaningless additive constant (the same relationship was obtained in [19, 20]).

We further take into account the Klein-Gordon equation (4) in the Einstein equations (5), and combine the  $tt$ - and  $xx$ -components to obtain a single relevant equation for the scale factor:

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = 0 , \quad (7)$$

which has a solution of the form

$$a(t) = a_0 e^{Ht} , \quad (8)$$

where  $a_0$  is a constant that can be absorbed in a redefinition of the spatial coordinates.

Once we know the solution for the scale factor, we further substitute its expression back into the Einstein equations, and combine the  $xx$ - and  $ww$ -components in order to get an equation for the warp factor  $A(w)$ :

$$A'' - A'^2 + H^2 = 0 . \quad (9)$$

This equation has the solution:

$$A(w) = \ln \left[ \frac{H}{b} \operatorname{sech} [H(w - w_0)] \right] , \quad (10)$$

where the integration constant  $b$  is related to the 5D cosmological constant as follows:

$$\Lambda_5 = 6b^2 , \quad (11)$$

which reveals the de Sitter nature of the 5D bulk space-time.

On the other hand, after taking into account the solution for the scale factor (8), and assuming the following *ansatz* for the scalar field:

$$\sigma(t, w) \sim u(t, w) = \epsilon(t) \chi(w) e^{-\frac{3}{2}A} , \quad (12)$$

the Klein-Gordon equation reduces to a couple of ordinary differential equations:

$$\chi'' - \left( \frac{3}{2}A'' + \frac{9}{4}A'^2 - \Omega^2 \right) \chi = 0 , \quad (13)$$

$$\ddot{\epsilon} + 3H\dot{\epsilon} + \Omega^2\epsilon = 0 , \quad (14)$$

where  $\Omega$  is an arbitrary constant.

The equation (13) can be regarded as a Schrödinger equation with the analog quantum mechanics potential:

$$V_{QM} = \frac{3}{2}A'' + \frac{9}{4}A'^2 . \quad (15)$$

Using the solution for the warp factor (10) we see that it represents a modified Pöschl-Teller potential of the form:

$$V_{QM}(w) = \frac{9}{4}H^2 - \frac{15}{4}H^2 \operatorname{sech}^2 [H(w - w_0)] . \quad (16)$$

Thus, the differential equation for  $\chi$  (13) turns out to be a known eigenvalue problem with a mixed spectrum (see [28], for instance). Namely, there are two bound states: a

ground state with  $\Omega = 0$  and another one with  $\Omega = \sqrt{2}H$ , separated by a mass gap determined by the asymptotic value:

$$V_{QM}(\infty) = \frac{9}{4}H^2, \quad (17)$$

as well as a continuum of KK states starting at  $\Omega = 3H/2$ .

In fact, the equation for  $\chi$  is precisely the eigenvalue equation for the transverse traceless KK metric fluctuations and massless scalar perturbations of the recently proposed de Sitter braneworld model [23, 29], where there is a mass gap in the graviton and scalar mass spectra of KK perturbations.

Thus, the equation for the relevant function  $\chi$  possesses a general solution:

$$\chi(w) = C_1 P_{3/2}^\mu [\tanh(H(w - w_0))] + C_2 Q_{3/2}^\mu [\tanh(H(w - w_0))] , \quad (18)$$

where  $P_{3/2}^\mu$  and  $Q_{3/2}^\mu$  are associated Legendre functions of first and second kind, respectively, grade  $\nu = 3/2$  and order

$$\mu = \sqrt{\frac{9}{4} - \frac{\Omega^2}{H^2}}. \quad (19)$$

Thus, the first discrete state is the ground state with  $\Omega = 0$ , order  $\mu = 3/2$  and

$$E_0 = -\frac{9}{4}H^2. \quad (20)$$

The explicit expression of this zero mode is given by:

$$\chi_0(w) = c_0 \operatorname{sech}^{3/2}[H(w - w_0)] , \quad (21)$$

where  $c_0$  is a normalization constant. On the other hand, the second bound state corresponds to an excited mode with  $\Omega = \sqrt{2}H$ , order  $\mu = 1/2$  and energy

$$E_1 = -\frac{1}{4}H^2, \quad (22)$$

and has the following form:

$$\chi_1(w) = c_1 \sinh[H(w - w_0)] \operatorname{sech}^{3/2}[H(w - w_0)] , \quad (23)$$

where  $c_1$  also is a normalization constant.

Turning back to the equation for  $\epsilon(t)$  (14) we see that it is the equation for a damped oscillator which has three different solutions depending on the kind of damping, i.e. on the relation between the constants  $H$  and  $\Omega$ : a). under-damping ( $\Omega^2 > 9H^2/4$ ), b). critical damping ( $\Omega^2 = 9H^2/4$ ), and c). over-damping ( $\Omega^2 < 9H^2/4$ ).

a). The solution for the under-damped case reads:

$$\epsilon(t) = Ce^{-\frac{3}{2}Ht} \sin(\omega t + \delta) \quad (24)$$

where the constant  $C$  denotes the oscillations amplitude, the parameter

$$\omega = \sqrt{\Omega^2 - \frac{9}{4}H^2} \quad (25)$$

is the un-damped frequency of the oscillations and  $\delta$  is an arbitrary phase constant. The constants  $C$  and  $\delta$  are determined by initial conditions. We can easily see that

these oscillations will exponentially decay to zero with time, a fact that translates into a vanishing metric function  $u \rightarrow 0$ , which leads in turn to an isotropic 5D metric with a 3-brane with de Sitter symmetry [23].

b). The solution for the critical damping case reads:

$$\epsilon(t) = e^{-\frac{3}{2}Ht} (\alpha t + \beta), \quad (26)$$

where  $\alpha$  and  $\beta$  are arbitrary constants determined by initial conditions. In this case we observe the same effect as in the under-damped one: the amplitude of the function  $\epsilon$  exponentially vanishes with time, making the metric function  $u$  disappear and yielding a de Sitter 3-brane embedded in an isotropic 5D de Sitter space-time.

c). Finally, the over-damped case possesses the following solution:

$$\epsilon(t) = e^{-\frac{3}{2}Ht} (\alpha e^{\tilde{\omega}t} + \beta e^{-\tilde{\omega}t}), \quad (27)$$

where

$$\tilde{\omega} = \sqrt{\frac{9}{4}H^2 - \Omega^2}, \quad (28)$$

and  $\alpha$  and  $\beta$  are arbitrary constants determined by initial conditions. Once again we see that the metric function  $u$  exponentially decays in time, leading to a completely isotropic 5D metric with a de Sitter 3-brane embedded in it and realizing a very natural isotropization mechanism.

Thus, we see that in the general case, the solution for the time evolution of the metric function  $u(t, w)$  expressed by (12) exponentially yields an isotropic 5D metric which possesses an embedded 3-brane with de Sitter symmetry under a wide class of initial conditions. It is worth noticing that together with the function  $u$ , the scalar field also exponentially disappears as a consequence of their proportionality, rendering a completely geometric de Sitter thick brane with very appealing features (see [23, 29] for details).

In order to understand in detail what happens to the anisotropy of the initial metric and give a consistent physical interpretation to this process, it is necessary to compute the Weyl tensor and its projection to the brane using, for example, the formalism of [16]. However, it is clear that the inflationary process that isotropizes the metric is due to the 4D cosmological constant (since it is equal to  $\Lambda_4 = 3H^2$ ), which is also responsible for the vanishing of the phantom-like bulk scalar field.

### 3. Conclusions

In this paper we have presented a novel mechanism of isotropization of an initially anisotropic 5D thick braneworld generated by a phantom-like scalar field minimally coupled to gravity. Under a wide class of suitable initial conditions the anisotropic braneworld evolves and exponentially isotropizes by itself, under the action of the 4D cosmological constant, while the scalar field disappears. Thus, the anisotropic energy of the 3-brane rapidly leaks into the bulk through the nontrivial components of the Weyl

tensor, leading to a less isotropic bulk. As a result we have an isotropic thick braneworld supported by pure curvature, where 4D gravity as well as other matter fields have been shown to be localized, the effective 4D Planck mass is finite and depending on the Hubble parameter, the 4D cosmological constant can adopt an infinitely small value without any fine-tuning to the 5D one, and the mass spectrum of KK gravitons displays a mass gap, an important feature from the phenomenological point of view; moreover, the corresponding corrections to Newton's law have also been computed [23, 29].

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